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MESA 101 | Debates del Marxismo contemporáneo en torno al "Marxismo Cuantitativo" y al "Problema de la transformación de valores a precios"

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Resumen

En el presente trabajo se exponen los avances recientes y el estado del debate en torno al problema de la transformación de valores a precios. El estudio expone los distintos trabajos en base a las contraposiciones entre soluciones estáticas (reproducción simple y ampliada) vs. dinámicas (Temporal Single System), la crítica ricardiana y la `nueva solución' de Foley y Duménil, y se propone un análisis crítico y englobador de estas teorías. Como se expone en el trabajo el problema puede resumirse a la conveniencia analítica y empírica para decidir que transformar en base a la igualdad de valores y precios y qué variables dejar que difieran. Esta conveniencia se debe juzgar en base a las potencialidades del modelo para explicar rasgos centrales del capitalismo en un paradigma marxista. Entre estos: (i) explicar la tasa de beneficio como resultado de la explotación del trabajo y extracción de la plusvalía; (ii) mostrar una conexión clara entre variables observadas y su contraparte en valores, (iii) explicar las leyes de movimiento del capitalismo (tendencia decreciente de la tasa de ganancia, crisis, etc.).

Palabras claves: transformación de valores a precios, precios de producción, teoría del valor

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1. Introduction

The history of the transformation problem led to an exciting and rich debate about the theoretical foundations of labor value theory. We refer the reader to the excellent reviews relevant for this paper of Foley (2000) and Loranger (2004), and we only present some highlights.

The neo-Ricardian solution consensus is based on solving a simple reproduction economy in which there is a dual system of labor values and prices of production. The transformation problem is defined in this set-up as the problem of finding a positive price vector of commodities and a scalar positive profit rate that fulfills simple reproduction, and how these variables relate to labor values. Since the work of von Bortkiewicz (1952), it has been generally accepted that two key value-price invariance postulates, that is, "Aggregate value = Aggregate price" and "Aggregate surplus value = Aggregate profits", cannot simultaneously hold except under exceptional circumstances, and thus "either aggregate labor is not the sole determinant of aggregate price, or aggregate unpaid labor is not the sole determinant of aggregate profit" (Mohun, 1994, p.394). Marxian surplus value is interpreted as the difference (in values and prices) between a wage bundle of commodities that guarantees reproduction of the labor force and net output. This solution, however, does not require any reference to values except for the so-called Fundamental Marxian Theorem (see the comprehensive study in Morishima, 1973) in which exploitation is a necessary condition for positive profits. The neo-Ricardian solution thus relies on the technical determination of social relations. Moreover, it concludes that only commodities that enter directly

or indirectly in the production of the wage bundle are necessary to determine the profit rate.

The 1980s `New Interpretation' (NI hereafter) approach appears as a rejection of the neo-Ricardian, based on the original works of Foley (1982, 1986, 2000) and Duménil (1983), and formalized by Lipietz (1982) and Glick and Ehrbar (1987) among others. This interpretation is based on the monetary expression of labor as the ratio of money value added to total labor expended in its production, which logically implies a wage rate given by the proportion of the net output that goes to workers. As argued by Foley (1982) "[w]orkers in capitalist society do not bargain for, or receive a bundle of commodities as payment for the labor power, they receive a sum of money, the money wage, which they are then free to spend as they wish" (p.43) and thus the neo-Ricardian solution imposes an unrealistic "unilateral causality" (Lipietz, 1982, p.75). The NI has the salient feature of imposing an invariance value-price equation in variable capital and in net output, in a simple reproduction economy.

Other Marxian approaches are in general sympathetic to the NI but they coincide in criticizing the simple reproduction scheme and emphasize the necessity of considering constant capital too as a measure invariant in values and prices. Wolff, Roberts and Callari (1984) postulate the equality of constant capital in labor values and prices. They do so on the basis of a different interpretation of values as the sum of price-valued constant capital and labor units (see Foley, 2000, p.31). Moseley (1993, 2000) states that "constant capital and variable capital do not have to be transformed from the value magnitudes to price magnitudes, because constant capital and variable capital are *not determined first as the value of the means of production and wage-goods* and then later determined as the price of these bundles of goods" (Moseley, 2000, p.303). The dynamic and non-equilibrium Temporal Single System (TSS) framework (see

the articles in Freeman and Carchedi, 1996) also emphasizes the necessity of considering a model where constant capital is not different in values and prices.

Once wages are set, the NI derives the profit rate from the price system only (see Moseley, 2000, pp.309-310), and thus the profit rate in labor values appears to be redundant. This is not entirely satisfactory as the profit rate is a key variable to explain capital distribution among sectors, accumulation, crisis and other factors in a capitalist society. An alternative to the NI, also in a reproduction scheme, is the `profit rate invariant' solution of Loranger (2004). This holds the equality of the profit rate in prices of production and values. This solution could be obtained from an invariant equation based on the aggregate components of capital, constant and variable jointly, and surplus value and profits. However, as shown in this paper, this may produce negative wages for some levels of exploitation, or positive wages with maximum exploitation, both of which are inconsistencies with no economic interpretation.

This paper constructs a new solution in a simple reproduction economy, where wages are well defined, and the profit rate is close (not identical) to the profit rate in values. It imposes two invariant equations in terms of both constant and variable capital, separately, to solve the price and value system. The key feature of commodity production in the Marxian analysis is that it is a result of capital exploitation of labor, and, as emphasized by both the NI and other contemporary approaches to the transformation problem, capital appears in the form of money, i.e. as advanced money to hire workers (variable capital) and to buy inputs or means of production (constant capital). The NI emphasizes the importance of an invariance equation for the former, while the alternative interpretations outlined above emphasize the necessity of an invariant equation for the latter (although not in the reproduction model). This paper imposes both of them simultaneously within the simple reproduction scheme.

This solution produces a profit rate that lies in between the NI and the labor-valued profit rate. For low values of the exploitation rate this solution delivers a profit rate closer to the labor-valued system one, while for high values of the exploitation rate, the profit rate becomes closer to the price system profit rate. The two invariance equations cannot maintain simultaneously equality of gross and net output. In fact, the proposed solution also lays in between: it varies from being closer to the gross output equality for low values of exploitation to proximity to the net output one.

The proposed transformation solution in this paper should be framed within the following two conditions.

First, we consider a value-and-price dual system simple reproduction static economy with no growth. The reproduction scheme transforms input prices according to the resulting prices of production, at their corresponding replacement costs, and does not consider them as given and with the same magnitude in values or prices. Thus, we retain the methodological determination of prices of production embodied in the simple reproduction scheme, that is, the rate of profit, wages and prices of production should be determined simultaneously given the technical conditions of production and social relations. Moreover, the transformation procedure should obtain prices and values per unit of commodity, and not aggregate magnitudes only. We also rule out joint production schemes and maintain a one-to-one match between commodities and sectors of production. This is done on the basis that joint production contains an implicit exchange of value within a sector (or firm), which avoids the market environment, and thus deviates from the traditional transformation problem (and may produce negative values, as in Steedman, 1977).

Second, we consider a reproduction scheme where real wages are not pre-determined as the value/price of a subsistence wage bundle. Instead, real wages are endogenously determined. We follow the NI and Loranger (2004) (among

others) in rejecting the necessity of defining a wage subsistence bundle. We thus allow the exploitation rate to be exogenously determined by both technical and social relations.

The rest of the paper is organized as follows. Section 2 introduces the basic notation and derives the value and price systems. Section 3 reviews the `New Interpretation' approach. Section 4 reviews Loranger's (2004) profit invariant solution. Section 5 presents the new capital invariant solution. Numerical examples are presented in section 6. Section 7 concludes.

2. Values and prices of production

Consider a simple reproduction economy with *n* commodities, each produced by a different industry or sector. Let A be a Leontieff $n \times n$ matrix with typical element a_{ij} that specifies the quantity of commodity *j* required to produce one unit of good *i*. Consider the $1 \times n$ vector ℓ specifying the number of labor units required to produce one unit of each commodity, $\ell_i,\ i = 1,2, \dots n.$ A and ℓ summarize the technology.

The $1 \times n$ vector v represents labor values in this economy, and they satisfy the equation

$$vA + \ell = v. \qquad (1)$$

Thus, assuming that A is indecomposable, which then implies that $(I - A)^{-1}$ is positive, we have that $v \gg 0$ and

$$v = \ell(I - A)^{-1}. \qquad (2)$$

This means that labor values of each unit of commodity i, v_i, $i = 1,2, \dots n$, are in fact proportional to the amount of labor units required for its production, taking into account the inputs that are required for the production of all commodities.

Let $c = vA$, b and s be $1 \times n$ vectors with the constant capital, variable capital and surplus value, respectively, in each industry, such that $\ell = s + b, v = c + s + b$. Thus, the labor components in ℓ can be divided into surplus value and variable capital, and the labor values v can be disaggregated into three components. In Marxian terminology this explicitly accounts for the fact that capital exploits labor and appropriates a portion of its product. A central concept in Marxian analysis is the surplus value rate (also defined as the exploitation rate), given by the ratio $\rho \equiv \frac{s}{b}$, that is often assumed to be the same across sectors. Define the appropriation rate $\alpha \in [0; 1]$ that corresponds to the value appropiated by capitalists in production, where by definition, $\alpha \equiv \frac{s}{s+b}$ and then $\rho = \frac{\alpha}{1-\alpha}$.

Consider now a capitalist economy with prices of production given by the $1 \times n$ vector p and with homogenous profit rate π and wage w. Prices of production are defined by the equation

$$(pA + w\ell)(1 + \pi) = p. \qquad (3)$$

Then we have

$$p = w\ell((1 + \pi)^{-1}I - A)^{-1}, \qquad (4)$$

which for a general positive price solution, $p \gg 0$, requires the price transformation matrix

$$\Pi(\pi) \equiv ((1 + \pi)^{-1}I - A)^{-1}$$

to be a positive matrix. Note that $(I-A)^{-1}$ being positive does not imply that $\Pi(\pi)$ also is, and this imposes a bound on the feasible profit rate π. Define $\pi^{MAX} = max\{\pi \geq 0 : \Pi(\pi) \gg 0\}$.

Define the gross product as the $n \times 1$ vector, Q, that contains each sector's gross product. Moreover, the net product is defined by $q \equiv (I-A)Q$. Define L as the aggregate labor force in this economy, and note that $vq \equiv \ell(I-A)^{-1}(I-A)Q = \ell Q = L$. The profit rate, as measured in values, is $\pi^{v}(\alpha) = \frac{\alpha L}{vA+(1-\alpha)L}$. The profit rate in prices is by definition $\pi = \frac{pq-wL}{pA+wL}$.

Note that in this system there are $n+2$ unknowns (n commodities' prices, the wage rate w and profit rate π) but only n equations. Different solutions to the transformation problem depend on the two additional equations to complete the system. Seton (1957) calls these invariance equations. A convex hull of the alternatives is that they should satisfy:

(I) Aggregate product in prices equals aggregate product in values. This could either be

(I') $pQ = vQ$ or

(I'') $pq = vq$.

(II) Aggregate surplus value equals aggregate profits, $\alpha L = pq - wL$.

(III) The profit rate in values equals the profit rate in prices, $\pi = \pi^{v}$.

The transformation problem could thus be defined as the solution to equations (2) and (4), under the invariance conditions (I) and/or (II) and/or (III). It should be noted that in the particular case of uniform organic composition of capital

across sectors, prices are proportional to values and thus (I)-(II)-(III) are simultaneously satisfied.

3. The `New Interpretation'

The `New Interpretation' (NI) is based on the original works of Foley (1982) and Duménil (1983), and formalized by Lipietz (1982). This approach justifies (I'') as a better alternative than (I') on the basis of avoiding double counting and interpreting capitalist exploitation as appropriation of value added. Moreover, it imposes a clever *numeraire*:

(IV) Wages are expressed in labor units, that is, $w^{ni} = 1 - \alpha$.

These choices are based on the interpretation of "the labor theory of value as the claim that the money value of the whole mass of the net production of commodities expresses the expenditure of the total social labor in a commodity-producing economy." (Foley, 1982, p.37). This monetary expression of labor time (MELT) has been criticized by Fine, Lapavitsas and Saad-Filho (2004) among others. We only consider the MELT as an invariant equation but avoid the discussion of the monetary theory behind it.

Define $\left(\pi^{ni}, w^{ni}, p^{ni}\right)$ as the NI profit rate, wage rate and prices of production, where $\left(\pi^{ni}, p^{ni}\right)$ is a solution to the price equation (4).

The NI guarantees that the net product $p^{ni}q$ is divided between workers and capitalist in accordance to α because $p^{ni}q = p^{ni}q - w^{ni}L + w^{ni}L = vq = L = \alpha L + (1-\alpha)L$, where the equality follows from (I''). Note that (II) is also satisfied.

However, the NI profit rate differs from the profit rates in values. That is,

$$\pi^{ni} = \frac{p^{ni}q - w^{ni}L}{p^{ni}AQ + w^{ni}L} = \frac{\alpha L}{p^{ni}AQ + (1-\alpha)L} \neq \pi^{v} = \frac{\alpha L}{vAQ + (1-\alpha)L},$$

unless $p^{ni}AQ = vAQ$, that is, aggregated inputs are valued equally in prices and in labor values. The NI approach derives the profit rate from the price system only (see Moseley, 2000, pp.309-310), and thus the profit rate in values may be redundant.

Define now $\left(\pi^{ni}(\alpha), w^{ni}(\alpha), p^{ni}(\alpha)\right)$ as the NI profit rate, wage rate and prices of production as a function of α. Note that $\pi^{ni}(\alpha) = \frac{\alpha L}{p^{ni}(\alpha)AQ + (1-\alpha)L} \leq \frac{\alpha L}{(1-\alpha)L} = \frac{\alpha}{1-\alpha}$ and then $\pi^{ni}(0) = \pi^{v}(0) = 0$. However, both profit rates diverge as $\alpha > 0$, and we could either have $\pi^{ni}(\alpha) < \pi^{v}(\alpha)$ or $\pi^{ni}(\alpha) > \pi^{v}(\alpha)$ for $\alpha \in (0,1]$.

4. The `profit rate invariant' solution

Loranger (2004) proposes an alternative solution imposing (I')-(II)-(III) as invariance equations and leaving the wage rate w endogenous. The same results could be obtained if the profit rate components', numerator and denominator, are assumed to be equal in values and prices of production. First, the numerator is condition (II) $\alpha L = pq - wL$, that is, aggregate surplus value equals aggregate profits, and it is consistent with the NI. Second, the denominator implies that `total costs' are equal in values and prices of production. This is equivalent to assuming that aggregate capital, both constant and variable, should be equalized to the aggregate value it represents:

(V) Constant capital plus variable capital are equal in values and in prices, that is, $vAQ + (1-\alpha)L = pAQ + wL$.

Note that imposing (II) and (V) is equivalent to imposing (I')-(II)-(III). However, this solution works only for the case with $\pi^{v} \leq \pi^{MAX}$, that is, if the profit rate in values correspond to prices of production that are positive.

Define $\left(\pi^v(\alpha), w^v(\alpha), p^v(\alpha)\right)$ as the profit rate, wage rate and prices of production as functions of α that arise as solution to the profit rate invariant transformation.

The resulting wages are not a claim on units of abstract labor as used in the NI, but simultaneously the required price of the labor force to make aggregate surplus value and advanced capital (constant plus variable) invariant as measured in values and prices of production. In the same way as commodities appear as a result of capital, the price of the labor force is also a commodity as seen through the eyes of the capitalist.

5. The `capital invariant' solution

Moseley (2000), Wolff, Roberts and Callari (1984), and the proponents of the TSS, among others, emphasize that capital only appears in a monetary form (i.e. prices). Condition (V), the invariance equation based on total capital, i.e. constant plus variable, is in fact motivated by this literature. However, in general, those that emphasize the monetary nature of capital are also against the necessity of the transformation as described throughout this paper. That is, the simple reproduction economy model, where inputs are transformed according to prices of production, should be abandoned, because there is no need to transform values into prices. This paper works within the simple reproduction economy as used by the NI, Loranger (2004) and the neo-Ricardian approach.

The bottom line is that capital itself should be made invariant in some way. In fact, condition (V) and Loranger's (2004) solution explicitly imposes this, although not in a fully satisfactory way. The NI approach also goes in this direction by taking into account that variable capital (and thus wages) takes a monetary form, and this is essential in the capitalist exploitation mechanism.

We propose a new solution to the transformation problem in this direction. If we combine conditions (IV) from NI and (V) from Loranger (2004) we obtain a new solution in which we are imposing that both constant and variable capital should be made invariant. (IV) and (V) imply that the following condition holds:

(V') Constant capital is equal as measured in values and in prices, that is, $vAQ = pAQ$.

This proposed solution is defined as `capital invariant' (CI hereafter), because the different components of capital, i.e. constant and variable, separately, are made invariant. As with any solution to the transformation problem, it has some desired and undesired (or unexpected) features. Condition (IV) guarantees that wages are well defined for the whole range of $\alpha \in [0,1]$, thus avoiding the issue of positive wages with $\alpha = 1$ and that of negative wages for some α. Condition (V) imposes that both profit rates, i.e. in prices or values, share the same denominator. Thus, because profits depend on exploitation, the profit rate in prices is connected to the profit rate in values, avoiding a total disconnection between the two as in the NI.

Note that there is no guaranteed equality in prices and values for gross or net ouputs, except for extreme values of the exploitation parameter. That is, gross output in prices and values are approximately equal for low values of α (as in Loranger's profit rate invariant solution), and net output in prices and values are approximately equal for both low and high values of α (as in the NI). But there is no global equality in output for $\alpha \in (0,1)$.

One feature of the CI transformation, similar to the NI and Loranger's solution, is that the real wage changes as a result of the transformation. This is a consequence of not using a fixed (subsistence) consumption bundle, i.e. as an implicit invariant measure of labor. Another feature of this solution is that the

price and value invariance is constructed in the average organic composition. That is, the ratio of variable capital to constant capital is invariant when aggregate magnitudes are considered: $\frac{(1-\alpha)L}{vAQ} = \frac{(1-\alpha)L}{p^{ci}(\alpha)AQ}$.

The proposed CI solution is implicitly taking the `eyes' of the capitalist. In capitalist production, embodied labor, from constant capital, and living labor, from variable capital, appear indistinguishable to the capitalist. For the capitalist, profit

"(...) springs from the productive process undertaken with the capital, that it therefore springs from the capital itself, because it is there after the production process, while it is not there before it. As for the capital consumed in production, the surplus-value seems to spring equally from all its different elements of value consisting of means of production and labor. For all these elements contribute equally to the formation of the cost-price. All of them add their values, obtaining as advanced capital, to the value of the product, and are not differentiated as constant and variable magnitudes of value." (Marx, 1894, *Capital*, Vol. III, ch.1, p.23)

Moreover, "The capitalist does not care whether it is considered that he advances constant capital to make a profit out of his variable capital, or that he advances variable capital to enhance the value of the constant capital, that he invests money in wages to raise the value of his machinery and raw materials, or that he invests money in machinery and raw materials to be able to exploit labor. Although it is only the variable portion of capital which creates surplus-values, it does so only if the other portions, the conditions of production, are likewise advanced. Seeing that the capitalist can exploit labor only by advancing constant capital and that he can turn his constant capital to good account only by advancing variable capital, he lumps them all together in his imagination, and much more so since

the actual rate of his gain is not determined by its proportion to the variable, but to the total capital, not by the rate of surplus-value, but by the rate of profit. And the latter, as we shall see, may remain the same and yet express different rates of surplus-value." (Marx, 1894, *Capital*, Vol. III, ch.2, p.27).

A solution to the Marxian transformation problem should be judged by the adequacy of its outcome to explain key features of capitalism in a Marxian paradigm. In particular, (i) if it describes capitalism in terms of exploitation of labor and shows a clear connection between key observable variables in prices, such as profit rates, and their corresponding counterpart in (labor) values; and (ii) if it helps in understanding the laws of motion of capitalism.

In terms of (i), the transformation solution should explain the origins of `non-labor income categories that are recognized by "vulgar economists", i.e. profits, interest, and rent' (Baumol, 1974, p.52). The transformation of values into prices addresses this analytically, and for this reason, any proposed solution should be considered as a tool to study certain features of the capitalist mode of production. In particular, the profit rate is a key variable to understand capital distribution among sectors, accumulation, crisis and other factors in a capitalist society. The NI has been proposed as the most salient solution in recent decades, although it has been criticized as not being able to explain the profit rate from labor value theory. Loranger (2004) proposed a transformation that relies on equalizing the price and value profit rates, but as shown in this paper, endogenous wages are not consistent for all levels of exploitation. As shown in this paper, both solutions are the result of a certain consensus in the literature in which capital should be made invariant in prices and values. This paper proposes a solution that makes an invariant transformation in both components of capital, constant and variable, and which maintains some features of the NI and some of Loranger's solution.

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